1 Epimorphisms and Monomorphisms

Definition 1.1 (Monomorphism)**.** $f : A \to B$ is a monomorphism if it has the property that for any $g, h : C \to A$, $fg = fh$ then $g = h$. (f is left cancellable)

Definition 1.2 (Epimorphism)**.** $f : A \to B$ is an epimorphism if it has the property that for any $g, h : B \to C$, $gf = hf$ then $g = h$. (f is right cancellable)

A decent intuition is that injective functions are monomorphisms, and surjective functions are epimorphisms.

Definition 1.3. A split monomorphism is an arrow with a left inverse.

Definition 1.4. A split epimorphism is an arrow with a right inverse.

And for both of these, we have the following:

Definition 1.5. If $r : X \to A$ and $s : A \to X$ is such that $rs = \mathrm{Id}_A$ then s is a splitting/section of r and r is a retraction of s. A is retract of X.

2 Initial and Terminal Objects

Definition 2.1. An object 0 is initial in a category if for any object x, there is a unique morphism into x, $0 \to x$.

Definition 2.2. An object 1 is terminal in a category if for any object x, there is a unique morphism from x into 1, $x \to 1$.

Initial and terminal objects are unique up to isomorphism, since if we have two initial objects, then we have an isomorphism which is unique, and similarly with terminal objects. Of course, this doesn't mean that there is just a single initial or terminal object.

3 Products of objects

A product diagram is an object P with maps $p_1 : P \to A$ and $p_2 : P \to B$ such that if we have another object X and maps $x_1 : X \to A$ and $x_2 : X \to B$, there is a unique $u : X \to P$ such that Figure 1 commutes. P is called a product of A and B

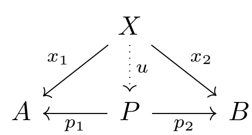

Figure 1: Product UMP Diagram

Products are unique up to isomorphism.

4 Hom sets

Definition 4.1. Let A, B be objects in a locally small category C. Then we have that $\mathrm{Hom}(A, B) = \{\, f \in \mathrm{Mor}\, C \mid f : A \to B \,\}$.

Read as the set of arrows/morphisms/homomorphisms from A to B.

If we have any other arrow $g : B \to C$, there is a function: $\mathrm{Hom}(A, g) : \mathrm{Hom}(A, B) \to \mathrm{Hom}(A, C)$ where $f \mapsto g \circ f$.

There is a functor $\mathrm{Hom}(A, -) : C \to \textbf{Sets}$ which is called the covariant representable functor of A.